Here is a list of vector spaces we have gone over in class:

- For $n \in \mathbb{Z}^+$, $\mathbb{R}^n$ is the n dimensional Euclidean space.
$$\mathbb{R}^n = \Big\{(a_1, a_2, a_3, \dots, a_n) \,|\, a_i \in \mathbb{R}, 1 \leq i \leq n\Big\}$$
- $F(\mathbb{R})$ is the set of all functions $f : \mathbb{R} \to \mathbb{R}$
- $P(\mathbb{R})$ is the set of all polynomials (with no restriction on the degree) with real coefficients
- $P_n(\mathbb{R})$ is the set of all polynomials of degree $\leq n$ with real coefficients
- $C(\mathbb{R}) = \{f : \mathbb{R} \to \mathbb{R} \,|\, f \text{ is continuous}\}$
- $C^1(\mathbb{R}) = \{f : \mathbb{R} \to \mathbb{R} \,|\, f \text{ is differentiable everywhere and } f'(x) \in C(\mathbb{R})\}$. The elements of $C^1(\mathbb{R})$ are called continuously differentiable functions.
- $C^2(\mathbb{R}) = \{f : \mathbb{R} \to \mathbb{R} \,|\, f \text{ is twice differentiable and } f''(x) \in C(\mathbb{R})\}$.
- $C^\infty(\mathbb{R}) = \{f : \mathbb{R} \to \mathbb{R} \,|\, f \text{ has derivatives of all orders}\}$
- $F([a, b])$ is the set of all functions $f : [a, b] \to \mathbb{R}$
- $C([a, b])$ is the set of all continuous functions $f : [a, b] \to \mathbb{R}$
- $M_{m \times n}(\mathbb{R})$ is the set of all matrices of size $m \times n$. This set is also denoted by $\mathbb{R}^{m \times n}$.

Comments:

- $P(\mathbb{R})$, $P_n(\mathbb{R})$, $C(\mathbb{R})$, $C^1(\mathbb{R})$, $C^2(\mathbb{R})$, $C^\infty(\mathbb{R})$ are all subspaces of $F(\mathbb{R})$.
- For each n, the vector space $P_n(\mathbb{R})$ is a subspace of $C^1(\mathbb{R})$.
- $C^1(\mathbb{R})$ is a subspace of $C(\mathbb{R})$ because differentiability implies continuity.
- $C^2(\mathbb{R})$ is a subspace of $C(\mathbb{R})$.
- $C([a, b])$ is a subspace of $F([a, b])$.
- $P(\mathbb{R}) \subset C^2(\mathbb{R}) \subset C^1(\mathbb{R}) \subset C(\mathbb{R})$.